

THEORY OF THE DETONATION OF WEAK CONDENSED EXPLOSIVES

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The possibilities of detonation taking place in a material characterized by a shock adiabat containing a sharp break (leading to a double shock-wave configuration) are examined. The range of possible velocities D of a self-sustaining detonation in the second shock wave is determined; D may be subsonic with respect to the original material. However, even for an arbitrarily low velocity of sound the range of subsonic D values above the break point on the adiabat is extremely limited: The minimum detonation velocity D_{\min} coincides (apart from a factor of 0.5-0.8) with the velocity of a longitudinal sound wave in the original material below the break point. This limitation with regard to D is associated with the formation of a wave of rarefaction in the reaction products. For $D < D_{\min}$ the shock wave of rarefaction reaches the Jouguet point and breaks the steady-state complex of the detonation wave. The results obtained are valid not only for weak, but also for powerful, explosive substances, if (by virtue of any kind of losses) low-velocity forms of detonation are realized in these materials.

The detonation of condensed low-power explosives involves certain singularities associated with the strength characteristics of the explosive materials (solid explosives), or with phase transformations of the first kind, leading to a break in the shock adiabat. Depending on the heat q of the irreversible chemical reaction, the intensity of the uniaxial compression σ^* (or pressure in the case of a liquid) in the shock wave may be either smaller or greater than the elastic limit (or the pressure at the onset of the phase transition). Subsequently, in order to be perfectly specific, we shall identify the break point on the shock adiabat with the elastic limit of a solid. The mutual disposition of the shock adiabat I and the detonation adiabat II for $\sigma < \sigma^*$ is illustrated in Fig. 1. From the practical point of view the case of $\sigma < \sigma^*$ is not very interesting, since any explosive suitable for arbitrarily long storage reacts very slowly in an elastic wave, and the real dimensions of the charge are always smaller than critical.

Other conditions being equal, a rise in the value of q in a substance with normal thermodynamic properties will correspond to a "higher" position of the detonation adiabat on the σv plane in a greater stress σ in the shock wave. The derivative $d\sigma/dq$ is continuous up to the point $q = q^*$, corresponding to the break on the shock adiabat. A slight further increase in q corresponds to a sharp change in σ from σ^* to values lying above point 3 (Fig. 1), and the creation of an anomalously high chemical peak. The rate of the ir-

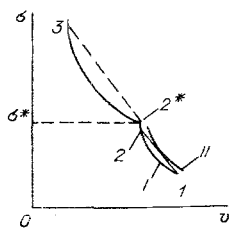


Fig. 1

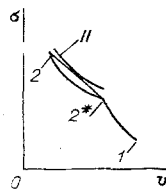


Fig. 2

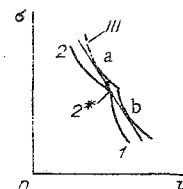


Fig. 3

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reversible chemical reaction also increases abruptly at this point; this creates a very severe instability of the plane front (leading edge) [1-3], and should lead to large-scale pulsations in the detonation wave at $q \approx q^*$. This form of detonation (with an anomalously large chemical peak) may be hard to produce under laboratory conditions for a number of practical reasons.

However, over a certain range of q values there is yet another possibility, namely, that a detonation-induced transformation of the material associated with the splitting of the shock wave in the latter may occur. During the rise in the chemical peak from point 2* to point 3 (Fig. 1), the shock wave propagating with respect to the original state I is unstable and decomposes into two shock waves [4-7]. The first ("elastic") wave converts the material from state 1 into state 2*. Further compression occurs in the second wave, i.e., the wave propagating with respect to state 2*. In the range of intensities 2*-3, including point 3 itself, the second wave moves more slowly than the first, so that the configuration of the two steady-state waves as a whole is nonstationary (transient). The comparatively high compressibility in the second wave leads to a considerable rise in temperature, which may be especially large at individual points in macroscopic inhomogeneities, for example, in the neighborhood of macroscopic defects, such as cracks and pores. Local heating of this kind creates favorable kinetic conditions for detonation in the second wave.

In the coordinate system in which the material behind the leading edge of the first shock wave is at rest, the self-sustaining, steady-state detonation in the second wave is described by the detonation adiabat II, characterized by an initial state 2* and the condition that the Rayleigh-Michelson straight line should be tangential to it (Jouguet condition, Fig. 2). Depending on the heat of reaction and the value of the break in the shock adiabat at point 2*, the rate of detonation in the second wave D may be either greater than or less than the velocity of sound in the original state 1. The value of D cannot, of course, be smaller than the propagation velocity of the weak wave of unilateral compression in state 2*, however slight the heat evolution. However, there is also a more severe limitation on the rate of detonation in the second wave, associated with the fact that (for thermodynamic and gasdynamic reasons) a self-sustaining detonation in the second wave is impossible if the heat evolution q is less than a certain minimum value $q_{\min}^{(2)}$. We may most readily convince ourselves of the impossibility of detonation for very low heat evolution in the second wave on the basis of the following qualitative considerations. For an infinitely small value of q the intensity of heat evolution in the second wave would be infinitely small and equal to $q\rho_1 D$ (here ρ_1 is the density of the original explosive). This would then by no means suffice to "feed" the first shock wave, the total energy \mathcal{E} of which increases in unit time by a finite amount

$$\frac{d\mathcal{E}}{dt} = \rho^* \left[E^* + \frac{(u^*)^2}{2} \right] (D^* - D).$$

Here D^* is the velocity of the first shock wave; u^* , E^* , and ρ^* are the mass velocity, the internal energy of unit mass, and the density of the material in the first wave, respectively.

In order to find the minimum heat evolution $q_{\min}^{(2)}$ for which the self-sustaining form of detonation becomes possible in the second wave, and also the associated minimum rate of detonation, let us consider the interaction of the subsonic zone of the reaction with the following wave of rarefaction. If the shock wave of the detonation complex does not split into two, this kind of interaction usually only takes place at the Jouguet point (adjacent to the region of supersonic flow) in a centered wave of rarefaction, which has no influence on the rate of propagation of the detonation [8, 9]. However, for a low heat evolution the isentropic line of the detonation products (Fig. 3) lies close to the original shock adiabat, and in the present case there will, accordingly, be an anomalous shape of the curve, containing a break point or a region in which the second derivative of the pressure with respect to volume is negative. For this kind of isentropic line it is quite impossible (with the aid of merely one centered wave of rarefaction) to satisfy the conditions corresponding to the self-sustaining mode of propagation of a detonation wave, bounded from the rear by a stationary wall or vacuum. According to the boundary conditions, on isentropic expansion from the Jouguet point the material passes through an anomalous region of the adiabat, in which [10, 11, 5] a shock wave of rarefaction is formed from the continuous wave. The amplitude of the shock wave of rarefaction in the vacuum increases until (for a specified original state of the compressed material) the velocity of the wave relative to the rarefied substance behind it becomes sonic. Further expansion of the material may occur in the continuous wave of rarefaction. Relative to the compressed gas the shock wave of rarefaction is supersonic. A steady-state detonation process may occur if the velocity of the shock wave of rarefaction does not exceed the rate of propagation of the steady-state detonation complex, determined by the Jouguet condition. Otherwise, the wave of rarefaction overtakes this complex and disrupts it. The desired minimum value of the heat evolution for detonation at the open end of a tube in contact with vacuum is deter-

mined by equating the velocities of the Jouguet detonation and the shock wave of rarefaction. This equation is represented graphically (Fig. 3) by the fact that the 2*-a section of the Rayleigh-Michelson straight line characterizing the detonation complex and the analogous segment a-b characterizing the shock wave of rarefaction form a single common straight line. For small relative deformations and a low heat evolution, the rarefaction shock adiabats drawn from differing initial states differ very little from one another. In Fig. 3 these adiabats are approximately represented in the form of a single curve similar in shape to the shock adiabats 1-2* and 2*-2 of the original substance, but lying above them.

The equal-velocity criterion determining the minimum heat evolution and the geometrical interpretation of this equation (Fig. 3) also applies to the case of detonation at the closed end of a tube, but subject to the condition that the lower tangent point b may actually be reached during the expansion of the detonation products, i.e., that the gas velocity has not become negative at point b (directed from the end into the tube). This imposes a certain limitation on the compression in the first shock wave, expressed by the inequality

$$u_a \geq \sqrt{(\sigma_a - \sigma_b)(v_b - v_a)}, \quad (1)$$

in which u_a is the velocity of the gas at the Jouguet point (v is the specific volume):

$$u_a = (v_1 - v^*) \sqrt{\frac{\sigma^* - \sigma_1}{v_1 - v^*}} + (v^* - v_a) \sqrt{\frac{\sigma_a - \sigma^*}{v^* - v_a}}. \quad (2)$$

Substituting (2) into (1) and remembering that

$$(\sigma_a - \sigma^*)/(v^* - v_a) = (\sigma_a - \sigma_b)/(v_b - v_a),$$

instead of (1) we obtain

$$(v_1 - v^*) \sqrt{\frac{\sigma^* - \sigma_1}{v_1 - v^*}} > (v_b - v^*) \sqrt{\frac{\sigma_a - \sigma^*}{v^* - v_a}}. \quad (3)$$

Inequality (3) is always satisfied if $v_b < v_1$, since, for the direction of the break in the shock adiabat under consideration $(\sigma^* - \sigma_1)/(v_1 - v^*) > (\sigma_a - \sigma^*)/(v^* - v_a)$.

In order to calculate the minimum detonation velocity in the second wave at the open end of the tube [or at the closed end subject to condition (3)], we express the shock adiabats of the original material in the neighborhood of the break point 2* in the form

$$\begin{aligned} \sigma^\pm &= \sigma^* \pm a^\pm(v - v^*) \pm b^\pm(v - v^*)^2, \\ b^\pm &> 0. \end{aligned} \quad (4)$$

The signs (\pm) in (4) indicate that the corresponding quantities have two values, the (+) sign relating to the shock adiabat above the break point 2* and the (-) sign to the shock adiabat 1-2*.

An analogous expansion for the shock rarefaction adiabat having its initial state at the Jouguet point may be written in the following way:

$$\begin{aligned} \bar{\sigma}^\pm &= \sigma^* \pm a^\pm(v - \bar{v}^*) \pm b^\pm(v - \bar{v}^*)^2 \pm f(v)q = \sigma^* \pm f(\bar{v}^*)q - A^\pm(v - \bar{v}^*) \pm B^\pm(v - \bar{v}^*)^2, \\ A^\pm &\equiv a^\pm \pm f'(\bar{v}^*)q, \\ B^\pm &\equiv b^\pm \pm f''(\bar{v}^*)q/2 > 0. \end{aligned} \quad (5)$$

Here the (+) and (-) signs refer to parts of the rarefaction shock adiabat lying above and below the break point \bar{v}^* , respectively. Expansion (5) follows from (4) on remembering that, for small relative deformations, the shock adiabat of the original substance and the rarefaction shock adiabat of the reaction products lie close to the corresponding isentropic curves, and that both these isentropes merge into one at $q=0$. The value of \bar{v}^* usually differs from v^* by a small amount proportional to q :

$$\bar{v}^* - v^* = Lq.$$

The value of the coefficient L is determined by the temperature dependence of the elastic limit (or the shape of the phase-equilibrium curve) and the equation of state of the reaction products. The function $f(v)$ is determined by the equation of state of the reaction products and the equation of the rarefaction shock adiabat having its initial state at the Jouguet point.

By equating the slopes of the tangents drawn from point 2* to the branches of the rarefaction adiabats $\bar{\sigma}^+$ and $\bar{\sigma}^-$, we may obtain an equation for the minimum value of q in the second wave. Omitting the

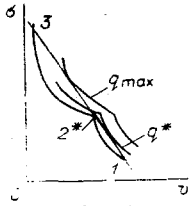


Fig. 4

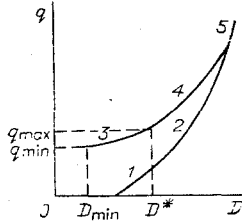


Fig. 5

simple calculations, we shall now present this equation, which was obtained by allowing for simply the lowest powers of the small quantity q :

$$A^+ - 2\sqrt{\alpha^+ B^+ q} = A^- - 2\sqrt{\alpha^- B^- q}, \quad (6)$$

$$\alpha^\pm \equiv f(v^*) - A^\pm L - B^\pm L^2 q \approx f(v^*) - A^\pm L > 0.$$

The latter inequality follows (for small q values) from the fact that $\bar{\sigma}^\pm > \sigma^\pm$ for any value of v .

Corresponding to the heat evolution $q = q_{\min}$ satisfying Eq. (6), we find the minimum detonation velocity D for the reaction in the second wave,

$$D_{\min} = v^* \sqrt{-A^+ + \frac{A^- - A^+}{1 + \sqrt{\alpha^- B^- / \alpha^+ B^+}}} + u^*. \quad (7)$$

Here u^* is the mass velocity behind the leading edge of the first wave:

$$u^* = D^*(v_1 - v^*)/v_1 \ll D_1.$$

For the small values of q considered here we have $A^\pm \approx a^\pm$. Transforming with due regard to this approximation from A^+ and A^- to the velocities c_+ and c_- of the longitudinal sound waves of the original material at the point 2^* (these velocities being respectively determined by allowing σ to approach σ^* from above and below), from Eq. (7) we obtain

$$D_{\min} = \sqrt{c_+^2 + \frac{c_-^2 - c_+^2}{1 + \sqrt{\alpha^- B^- / \alpha^+ B^+}}} + u^*. \quad (8)$$

It follows from Eq. (8) that, for the direction of the break in the shock adiabat under consideration, i.e., for $c_+ < c_-$, the minimum detonation velocity in the sound wave is greater than sonic,

$$D_{\min} > c_+.$$

The lowest value of the minimum detonation velocity in the second wave $D_{\min} = D_{\min}^0$ is reached when the properties of the original substance are such that the velocity of sound above the break point is very low $(c_+/c_-)^2 \rightarrow 0$. Then we have

$$D_{\min}^0 = \frac{c_-}{\sqrt{1 + \sqrt{\alpha^- B^- / \alpha^+ B^+}}} + u^* \approx \frac{c_-}{\sqrt{1 + \sqrt{\alpha^- B^- / \alpha^+ B^+}}}. \quad (9)$$

We note that c_- is the maximum rate of propagation of longitudinal sound vibrations in an elastically deformed material.

The coefficients B^+ and B^- of expansion (5) are usually quantities of the same order. The same is true of the coefficients α^+ and α^- . The possible quantitative difference between $\alpha^+ B^+$ and $\alpha^- B^-$ is furthermore greatly "smoothed" in the double extraction of the root in Eqs. (8) and (9). Substitution of $\alpha^+ B^+ = \alpha^- B^-$ in (9) gives

$$D_{\min} \approx c_- / \sqrt{2}. \quad (10)$$

We see from the structure of Eqs. (7)-(9) that varying the ratio $\alpha^- B^- / \alpha^+ B^+$ over a very wide range (e.g., 0.1-10) and allowing for the error in the approximation $A^\pm \approx a^\pm$ employed in deriving (8) and (9) only changes the result expressed in Eq. (10) by a factor very close to unity.

Apart from the foregoing lower limit, there is, of course, an upper limit to the detonation velocity in the second wave. This limit coincides with the maximum velocity of the "elastic" shock wave D^* . For $D > D^*$ the second wave overtakes the first, and corresponding to the steady-state classical one-dimensional detonation we have the ordinary detonation complex with a single shock wave.

The upper limit of heat evolution q_{\max} in the second wave corresponding to the limit $D = D_{\max}$ is greater than the heat evolution q^* for detonation in the first ("elastic") wave of maximum amplitude $\sigma = \sigma^*$. The difference between q_{\max} and q^* is the greater the smaller the maximum elastic deformation of the original substance. The rates of detonation for $q = q_{\max}$ and $q = q^*$ are the same, and equal to D^* , but the intensities of the shock waves are substantially different. This ambiguity in the form of dependence of q on the detonation velocity is associated with the break or bend in the detonation adiabat. The possibility

that the same Rayleigh–Michelson straight line may touch the two detonation adiabats is illustrated in Fig. 4. The ambiguous dependence of q on the velocity of the detonation wave D is illustrated schematically in Fig. 5. The numbers refer to parts of the $q(D)$ curves corresponding to the following modes of detonation discussed in the preceding paragraphs: 1) detonation within the range $q < q^*$ in the first shock wave; 2) detonation with an anomalously large chemical peak; 3) detonation in the second shock wave; 4) detonation with a normal chemical peak; 5) detonation for q values each of which corresponds to only one Jouguet point.

In conclusion, we note that the foregoing laws relate not only to weak, but also to strong, explosives, if, under certain specific conditions, the so-called slow modes of detonation are realized in the latter (see [12–15] and the literature cited in those papers). In a number of powerful condensed explosives we encounter [14] subsonic modes of detonation (relative to the velocity of longitudinal sound waves in the original material). Apart from a factor close to unity, the detonation velocities then agree with the values deduced from Eq. (10).

LITERATURE CITED

1. K. I. Shchelkin, "One-dimensional detonation instability," Dokl. Akad. Nauk SSSR, 160, 1144 (1965).
2. R. M. Zaidel' and Ya. B. Zel'dovich, "One-dimensional instability and attenuation of detonation," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 6, 59 (1963).
3. J. Erpenbeck, "Structure and stability of the square wave detonation," in: Ninth International Symposium on Combustion, New York (1963), p. 442.
4. D. Bancroft, E. Peterson, and S. Minshall, "Polymorphism of iron at high pressure," J. Appl. Phys., 27, No. 3, 291 (1956).
5. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], Fizmatgiz, Moscow (1963).
6. N. M. Kuznetsov, "Break in the shock adiabat during a phase transition of the first kind," Dokl. Akad. Nauk SSSR, 155, No. 1, 156 (1964).
7. N. M. Kuznetsov, "Structure of shock waves in the case of phase transitions of the first kind," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5, 140 (1964).
8. Ya. B. Zel'dovich and A. S. Kompaneets, Theory of Detonation [in Russian], Gostekhizdat, Moscow (1955).
9. L. D. Landau and E. M. Lifshits, Mechanics of Continuous Media [in Russian], GITTL (1953).
10. W. E. Drummond, "Multiple shock production," J. Appl. Phys., 28, 998 (1957).
11. A. G. Ivanov and S. A. Novikov, "Shock waves of rarefaction in iron and steel," Zh. Éksp. i Teor. Fiz., 40, 1880 (1961).
12. A. A. Sulimov, A. V. Obmenin, A. I. Korotkov, and P. I. Suslyabin, "Low-velocity detonation of high-density solid explosive charges," in: Transactions of the Third All-Union Conference on Combustion and Explosion [in Russian], Nauka, Moscow (1972), p. 464.
13. V. K. Bobolev, A. V. Dubovik, I. A. Karpukhin, and V. V. Rybakov, "Propagation of an explosion in thin layers of solid explosives," Fiz. Goreniya i Vzryva, 3, 331 (1969).
14. A. F. Belyaev, V. K. Bobolev, A. I. Korotkov, A. A. Sulimov, and S. V. Chuiko, Combustion-to-Explosion Transition of Condensed Systems [in Russian], Nauka, Moscow (1973).
15. N. M. Kuznetsov, "Ambiguity and stability of detonation conditions in a limited medium," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 1, 45 (1968).